B.Sc. Part III (Hons) 7th Paper

Diff. Eqns (contd.)

Q. Solve $(x+1)\frac{d^2y}{dx^2} - 2(x+3)\frac{dy}{dx} + (x+5)y = e^x$

Soln The given equation

$$(x+1)\frac{d^2y}{dx^2} - 2(x+3)\frac{dy}{dx} + (x+5)y = e^x$$

$$\Rightarrow \frac{d^2y}{dx^2} - 2\left(\frac{x+3}{x+1}\right)\frac{dy}{dx} + \frac{x+5}{x+1}y = \frac{e^x}{x+1}$$

which is of the form

$$y'' + Py' + Qy = R.$$

$$\therefore P = -2\left(\frac{x+3}{x+1}\right), \quad Q = \frac{x+5}{x+1}, \quad R = \frac{e^x}{x+1} \quad \text{---(1)}$$

Now $$1 + P + Q = 1 - \frac{2(x+3)}{x+1} + \frac{x+5}{x+1}$$

$$= \frac{x+1-2x-6+x+5}{x+1} = 0.$$

i.e. $1+P+Q = 0.$

$\Rightarrow$ $u = e^x$ is a part of CF of the given eqn.

Let the general solution be $y = uv$.

So v is given by

$$\frac{d^2v}{dx^2} + \left(P + \frac{2}{u}\frac{du}{dx}\right)\frac{dv}{dx} = \frac{R}{u}$$

$$\Rightarrow \frac{d^2v}{dx^2} + \left[-\frac{2(x+3)}{x+1} + 2e^{-x}e^x\right]\frac{dv}{dx} = \cancel{\frac{e^x}{(x+1)e^x}}$$

$$\Rightarrow \frac{d^2v}{dx^2} + \left[2 - \frac{2(x+3)}{x+1}\right]\frac{dv}{dx} = \frac{1}{x+1}$$

$$\Rightarrow \frac{d^2v}{dx^2} - \frac{4}{x+1}\frac{dv}{dx} = \frac{1}{x+1} \quad \text{---(2)}$$

Put $\frac{dv}{dx} = z \Rightarrow \frac{d^2v}{dx^2} = \frac{dz}{dx}$.

So (2) $\Rightarrow \frac{dz}{dx} - \frac{4}{x+1} z = \frac{1}{x+1}$ which is a linear eqn.

$$\therefore IF = e^{-\int \frac{4}{x+1}dx} = e^{-4\log(x+1)} = \frac{1}{(x+1)^4}$$

$\therefore$ soln is given by

$$z \times \frac{1}{(x+1)^4} = \int \frac{1}{(x+1)} \cdot \frac{1}{(x+1)^4} dx$$

$$\Rightarrow \frac{z}{(x+1)^4} = \int (x+1)^{-5} dx \Rightarrow z(x+1)^{-4} = \frac{(x+1)^{-4}}{-4} + k_1$$

$$\Rightarrow z = \frac{-1}{4} + k_1 (x+1)^4$$

$$\Rightarrow \frac{dv}{dx} = -\frac{1}{4} + k_1 (x+1)^4.$$

$\Rightarrow dv = -\frac{1}{4}dx + k_1 (x+1)^4 dx$. Integrating we get

$$v = -\frac{x}{4} + k_1 \frac{(x+1)^5}{5} + k_2 \quad \text{---(3)}$$

Hence, $y = uv$ is the required solu

where $u = e^x$ and v is given by eq (3).